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**A TWO-STEP FIRST-DIFFERENCE ESTIMATOR FOR  
A PANEL DATA TOBIT MODEL**

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## **ABSTRACT**

This study formulates an easy-to-use two-step first-difference estimator for a panel data Tobit model. In the first step a bivariate Probit is estimated, using all observations. These estimates are used to construct correction terms that are added to the first-difference equation. This equation is estimated by Least Squares on a sub-sample of observations for which the dependent variable is positive in both periods. Most of this study is concerned with the derivation of the corrections terms.

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## I INTRODUCTION

Kalwij (2003) develops a Maximum Likelihood estimator based on taking first-differences of the equation of interest for a panel data Tobit model with unit-specific effects that are allowed to correlate with explanatory variables in pre-specified way.<sup>1</sup> This latter aspect of the model is often referred to as the Conditional Mean Independence assumption (Wooldridge, 1995). The advantage of Kalwij's approach is that it yields parameter estimates that are less sensitive to a specific parameterisation of the individual specific effects than when using a standard panel data Tobit estimator. The disadvantage of his estimator is that it is difficult to use in practice since it demands programming of the likelihood contributions, which are non-standard. For this reason the main objective of this study is to construct an easy-to-use two-step estimator for a panel data Tobit model based on first-differences. For this purpose section 2 derives the Moment Generating Function (MGF) of a random variable that is defined as the difference of two censored normal random variables. Based on this result, section 3 formulates the two-step first-difference estimator for a panel data Tobit model.

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<sup>1</sup> Honoré (2002) provides an excellent survey on the estimation of nonlinear models with panel data and Lee (2001) provides a discussion of the advantages of first-differences estimators in panel data selection models.





## 2 THE MOMENT GENERATING FUNCTION (MGF) OF THE DIFFERENCE OF TWO CENSORED NORMAL RANDOM VARIABLES

$X$  and  $Y$  are two jointly distributed normal random variables:

$$\begin{pmatrix} X \\ Y \end{pmatrix} \sim N \left( \begin{pmatrix} \mu_X \\ \mu_Y \end{pmatrix}, \begin{pmatrix} \sigma_X^2 & \rho\sigma_X\sigma_Y \\ \rho\sigma_X\sigma_Y & \sigma_Y^2 \end{pmatrix} \right) \quad (1)$$

The density function of  $(X, Y)$  is the bivariate normal density function and is denoted by  $f_{XY}(x, y)$ . In the remainder of this study  $\phi(\cdot)$  denotes the standard normal density function,  $\Phi(\cdot)$  the standard normal distribution function and  $\Phi^2(\cdot, \cdot; \rho)$  the bivariate standard normal distribution function with a correlation coefficient  $\rho$  ( $\equiv \rho_{XY}$ , subscript  $XY$  is dropped).

$X$  is censored for values below  $a$  and  $Y$  is censored for values below  $b$ . The density function of the random variable  $Z = X - Y$  conditional on  $X > a$  and  $Y > b$  is given by (see Kalwij, 2003):

$$f_{Z|X>a, Y>b}(z | X > a, Y > b) = \frac{\frac{1}{\sigma_Z} \phi\left(\frac{z - \mu_Z}{\sigma_Z}\right) \Phi\left(-\frac{(\max\{a, b + z\} - \mu_{X|Z})}{\sigma_{X|Z}}\right)}{\Phi^2\left(\left(\frac{-a - \mu_X}{\sigma_X}\right), \left(\frac{-b - \mu_Y}{\sigma_Y}\right); \rho\right)}, \quad (2)$$

where  $\mu_Z = \mu_X - \mu_Y$ ,  $\sigma_Z^2 = \sigma_X^2 + \sigma_Y^2 - 2\rho\sigma_X\sigma_Y$ ,  $\rho_{XZ} = \frac{\sigma_X - \rho\sigma_Y}{\sigma_Z}$ ,

$$\mu_{X|Z} = \mu_X + \rho_{XZ} \frac{\sigma_X}{\sigma_Z} (z - \mu_Z), \text{ and } \sigma_{X|Z}^2 = (1 - \rho_{XZ}^2) \sigma_X^2.$$

The Moment Generating Function of the random variable  $Z = X - Y$  conditional on  $X > a$  and  $Y > b$  is given by (see Appendix A):

$$e^{t\mu_Z + \frac{1}{2}\sigma_Z^2 t^2} \frac{\int_{-\infty}^{+\infty} \phi(s) \Phi\left(-\frac{(\max\{a, (b + \mu_Z + \sigma_Z(s + t\sigma_Z))\} - \mu_X - \frac{\rho_{XZ}(s + t\sigma_Z)}{\sqrt{1 - \rho_{XZ}^2}})}{\sigma_X \sqrt{1 - \rho_{XZ}^2}}\right) \partial s}{\Phi^2\left(\left(\frac{-a - \mu_X}{\sigma_X}\right), \left(\frac{-b - \mu_Y}{\sigma_Y}\right); \rho\right)}. \quad (3)$$

The expectation of  $Z = X - Y$  conditional on  $X > a$  and  $Y > b$  is obtained by taking the derivate of the MGF of equation (3) with respect to  $t$  and evaluating this function in  $t=0$ . This yields (see Appendix B):

$$\begin{aligned}
\mu_{Z|X>a,Y>b} &= \left[ \Phi_2 \left( \left( \frac{-a-\mu_X}{\sigma_X} \right), \left( \frac{-b-\mu_Y}{\sigma_Y} \right); \rho \right) \right]^{-1} \times \left[ \mu_Z \int_{-\infty}^{\frac{a-b-\mu_Z}{\sigma_Z}} \phi(s) \Phi \left( \frac{\frac{a-\mu_X}{\sigma_X} - \rho_{XZ}s}{\sqrt{1-\rho_{XZ}^2}} \right) \partial s \right. \\
&+ \left. \mu_Z \int_{\frac{a-b-\mu_Z}{\sigma_Z}}^{+\infty} \phi(s) \Phi \left( \frac{\frac{b-\mu_Y + \sigma_Z s}{\sigma_Y} - \rho_{YZ}s}{\sqrt{1-\rho_{YZ}^2}} \right) \partial s \right. \\
&+ \left. (\sigma_X - \rho\sigma_Y) \phi \left( \frac{a-\mu_X}{\sigma_X} \right) \Phi \left( \frac{\rho \left( \frac{a-\mu_X}{\sigma_X} \right) - \left( \frac{b-\mu_Y}{\sigma_Y} \right)}{\sqrt{1-\rho^2}} \right) \right. \\
&+ \left. (\rho\sigma_X - \sigma_Y) \phi \left( \frac{b-\mu_Y}{\sigma_Y} \right) \Phi \left( \frac{\rho \left( \frac{b-\mu_Y}{\sigma_Y} \right) - \left( \frac{a-\mu_X}{\sigma_X} \right)}{\sqrt{1-\rho^2}} \right) \right]. \quad (4)
\end{aligned}$$

### 3 A TWO STEP FIRST-DIFFERENCE ESTIMATOR A PANEL DATA TOBIT MODEL

The model of interest is formulated as follows, assuming two time periods and N individuals:

$$y_{it}^* = x_{it}\beta + \alpha_i + \varepsilon_{it}, \quad (5)$$

$$y_{it} = \max(0, y_{it}^*) \quad t = \{1, 2\}, i = \{1, \dots, N\}.$$

This is a panel data Tobit model and is a straightforward extension of the standard Tobit model (Tobin, 1958) by including an additional dimension (time or individuals). The individual is indexed by  $i$  and the time period by  $t$ .  $x_{it}$  is a  $(1 \times K)$  vector of exogenous variables,  $\beta$  is a  $(K \times 1)$  vector of the parameters of interest and  $\alpha_i$  is an unobserved individual specific effect that may be correlated with  $x_{it}$ . The latent dependent variable is censored at zero and only  $y_{it}$  is observed. The error term  $\varepsilon_{it}$  is assumed to be a normal random variable with mean zero and variance  $\sigma_i^2$ ,  $\varepsilon_{it} \sim N(0, \sigma_i^2)$ , and is allowed to be serially correlated.

Taking the first difference of equation (5) to eliminate the individual specific effects from the main equation yields the following model:

$$\Delta y_i^* = \Delta x_i \beta + \eta_i \quad (6)$$

$$\Delta y_i = \begin{cases} \Delta y_i^* & \text{if } y_{i1}^* > 0 \text{ and } y_{i2}^* > 0 \\ 0 & \text{if } y_{i1}^* \leq 0 \text{ and } y_{i2}^* \leq 0 \\ y_{i2}^* & \text{if } y_{i1}^* \leq 0 \text{ and } y_{i2}^* > 0 \\ -y_{i1}^* & \text{if } y_{i1}^* > 0 \text{ and } y_{i2}^* \leq 0 \end{cases} \quad (7)$$

Where  $\Delta y_i^* = y_{i2}^* - y_{i1}^*$ ,  $\Delta x_i = x_{i2} - x_{i1}$  and  $\eta_i = \varepsilon_{i2} - \varepsilon_{i1}$ . The correlation coefficient of  $\varepsilon_{i1}$  and  $\varepsilon_{i2}$  is denoted by  $\rho$ . Given the distributional assumptions:  $\eta_i \sim N(0, \sigma_\eta^2)$  with  $\sigma_\eta^2 = \sigma_1^2 - 2\rho\sigma_1\sigma_2 + \sigma_2^2$ . One can obtain Maximum Likelihood Estimates of the parameters of interest using the density function as formulated in equation (2) (see Kalwij, 2003). An alternative is to use a two-step estimator in the spirit of Rochina-Barrachina (1999) who develops a two-step estimator for panel data selection models. For the special case of a panel data Tobit model this two-step procedure is outlined underneath.

Considering the model given by equation (6) and the nonzero observations on the dependent variable in both periods, the expectation of  $\Delta y_i$  conditional on  $y_{i1} > 0$  and  $y_{i2} > 0$  is

$$E[\Delta y_i \mid y_{i1} > 0, y_{i2} > 0] = \Delta x_i \beta + E[\eta_i \mid \varepsilon_{i1} > -x_{i1} \beta - \alpha_i, \varepsilon_{i2} > -x_{i2} \beta - \alpha_i]. \quad (8)$$

Using equation (3) of Section 2 the expectation at the RHS of equation (8) is written as follows:

$$E[\eta_i \mid \varepsilon_{i1} > -x_{i1} \beta - \alpha_i, \varepsilon_{i2} > -x_{i2} \beta - \alpha_i] = \pi_2 \Lambda_2(M_{i1}, M_{i2}, \rho) - \pi_1 \Lambda_1(M_{i1}, M_{i2}, \rho) \quad (9)$$

With

$$\pi_1 = (\sigma_1 - \rho \sigma_2) \quad (10)$$

$$\pi_2 = (\sigma_2 - \rho \sigma_1) \quad (11)$$

$$M_{i1} = (x_{i1} \beta + \alpha_i) / \sigma_1 \quad (12)$$

$$M_{i2} = (x_{i2} \beta + \alpha_i) / \sigma_2 \quad (13)$$

$$\Lambda_1(M_{i1}, M_{i2}, \rho) = \frac{\phi(M_{i1}) \Phi((M_{i2} - \rho M_{i1}) / \sqrt{1 - \rho^2})}{\Phi^2(M_{i1}, M_{i2}; \rho)} \quad (14)$$

$$\Lambda_2(M_{i1}, M_{i2}, \rho) = \frac{\phi(M_{i2}) \Phi((M_{i1} - \rho M_{i2}) / \sqrt{1 - \rho^2})}{\Phi^2(M_{i1}, M_{i2}; \rho)} \quad (15)$$

The standard normal distribution function is denoted by  $\Phi(\cdot)$ , the standard normal density function by  $\phi(\cdot)$  and the bivariate standard normal distribution function is denoted by  $\Phi^2(\cdot)$ . The correction terms are now added and subtracted from equation (6):

$$\Delta y_i = \Delta X_i \beta - \pi_2 \Lambda_2(M_{i1}, M_{i2}, \rho) + \pi_1 \Lambda_1(M_{i1}, M_{i2}, \rho) + \xi_i. \quad (16)$$

with

$$\xi_i = \pi_2 \Lambda_2(M_{i1}, M_{i2}, \rho) - \pi_1 \Lambda_1(M_{i1}, M_{i2}, \rho) + \eta_i.$$

$\xi_i$  is an error term with expectation zero (conditional on  $y_{i1} > 0$  and  $y_{i2} > 0$ ). If the correction terms are known then equation (16) can be estimated using least squares on the sample of individuals for whom the dependent variable is positive in both periods. As can be seen from eqs (10), (11) and (16), similar to the cross-sectional case, the proposed two-step estimator for a panel data Tobit model based on first-differences is a special case of the two-step first-difference estimator for panel data selection models as formulated in Rochina-Barrachina (1999).

The two-step estimation procedure is implemented as follows. In the first step we estimate the following bivariate Probit model:

$$y_{i1}^* = x_{i1} \beta + \alpha_i + \varepsilon_{i1}, \quad (17)$$

$$y_{i2}^* = x_{i2}\beta + \alpha_i + \varepsilon_{i2}, \quad (18)$$

$$I_{i1} = \begin{cases} 0 & \text{if } y_{i1}^* \leq 0 \\ 1 & \text{if } y_{i1}^* > 0 \end{cases}, \quad (20)$$

$$I_{i2} = \begin{cases} 0 & \text{if } y_{i2}^* \leq 0 \\ 1 & \text{if } y_{i2}^* > 0 \end{cases}. \quad (21)$$

We use the Mean Conditional Independence assumption of Wooldridge (1995) to deal with the unobserved individual specific effect  $\alpha_i$ . For this study it is only relevant that this approach essentially models the unobserved individual specific effect as a pre-specified function of observed characteristics and a random individual specific error term, say  $\alpha_i = h(x_{i1}, x_{i2})\gamma + \mu_i$ , with  $\mu_i \sim N(0, \sigma_\mu^2)$ . The parameter  $\gamma$  is an additional parameter of the model to be estimated. A popular choice for  $h(x_{i1}, x_{i2})$  is to take the average over time. The new residuals in equations (17) and (18) are  $u_{i1} = \mu_i + \varepsilon_{i1}$  and  $u_{i2} = \mu_i + \varepsilon_{i2}$  with variances  $\sigma_{u,1}^2 = \sigma_\mu^2 + \sigma_1^2$  and  $\sigma_{u,2}^2 = \sigma_\mu^2 + \sigma_2^2$ , respectively. The correlation between  $u_{i1}$  and  $u_{i2}$  is denoted by  $\rho_{12}$ . Using Maximum Likelihood for estimating the bivariate Probit model yields estimates for

$$\beta_1 = \frac{\beta}{\sigma_{u,1}}, \gamma_1 = \frac{\gamma}{\sigma_{u,1}}, \beta_2 = \frac{\beta}{\sigma_{u,2}}, \gamma_2 = \frac{\gamma}{\sigma_{u,2}} \text{ and } \rho_{12}.$$

In the second step we construct the correction terms using  $\hat{M}_{i1} = x_{i1}\hat{\beta}_1 + h(x_1, x_2)\hat{\gamma}_1$ ,  $\hat{M}_{i2} = x_{i2}\hat{\beta}_2 + h(x_1, x_2)\hat{\gamma}_2$  and  $\hat{\rho}_{12}$ . Next, the correction terms are substituted in equation (16):

$$\Delta y_i = \Delta X_i \beta - \pi_2^* \Lambda_2(\hat{M}_{i1}, \hat{M}_{i2}, \hat{\rho}_{12}) + \pi_1^* \Lambda_1(\hat{M}_{i1}, \hat{M}_{i2}, \hat{\rho}_{12}) + \xi_i \quad (22)$$

This equation is estimated using Least Squares on a sample of individuals for which the dependent variable is positive in both periods ( $y_{i1} > 0$  and  $y_{i2} > 0$ ). The auxiliary parameters  $\pi_1^*$  and  $\pi_2^*$  are defined as  $\pi_1^* = (\sigma_{u,1} - \rho_{12}\sigma_{u,2})$  and  $\pi_2^* = (\sigma_{u,2} - \rho_{12}\sigma_{u,1})$ .



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## APPENDIX A: MOMENT GENERATING FUNCTION

The moment generating function of the random variable  $Z$  given  $X > a$  and  $Y > b$  is defined as:

$$m(t) = \frac{\int_{-\infty}^{+\infty} e^{tz} \int_{\max\{a, b+z\}}^{\infty} f_{XZ}(x, z) \partial x}{\int_{-\infty}^{+\infty} \int_a^{\infty} \int_b^{\infty} f_{XY}(x, y) \partial y \partial x} \partial z \quad (\text{A1})$$

$$m(t) = \frac{\int_{-\infty}^{+\infty} e^{tz} \left[ \int_{\max\{a, b+z\}}^{\infty} f_{XZ}(x, z) \partial x \right] \partial z}{\int_a^{\infty} \int_b^{\infty} f_{XY}(x, y) \partial y \partial x} \quad (\text{A2})$$

The denominator of equation (8) can be written as:

$$\int_a^{\infty} \int_b^{\infty} f_{XY}(x, y) \partial y \partial x = \Phi_2 \left( \frac{-(a - \mu_X)}{\sigma_X}, \frac{-(b - \mu_Y)}{\sigma_Y}, \rho \right)$$

Where  $\Phi_2(\cdot)$  denotes the bivariate standard normal distribution function. The numerator of equation (A2) can be written as:

$$\int_{-\infty}^{+\infty} \left[ \int_{\max\{a, b+z\}}^{\infty} e^{tz} f_{XZ}(x, z) \partial x \right] \partial z \quad (\text{A3})$$

To simplify the integral we substitute

$$u = \frac{x - \mu_X}{\sigma_X} \quad \text{and} \quad v = \frac{z - \mu_Z}{\sigma_Z}. \quad (\text{A4})$$

Furthermore, to adjust the borders of integration we define:

$$c_1(v) = \frac{a - \mu_X}{\sigma_X} \quad \text{and} \quad c_2(v) = \frac{b + (\mu_Z + \sigma_Z v) - \mu_X}{\sigma_X} \quad (\text{A5})$$

Now (A3) becomes:

$$\int_{-\infty}^{+\infty} \left[ \int_{\max\{c_1(v), c_2(v)\}}^{\infty} e^{t(\mu_Z + \sigma_Z v)} \phi_2(u, v; \rho_{XZ}) \partial u \right] \partial v, \quad (\text{A6})$$

where  $\phi_2(\cdot, \cdot; \cdot)$  denoted the bivariate standard normal distribution. Hence, equation (A6) is rewritten as follows:

$$e^{t\mu_Z} \int_{-\infty}^{+\infty} \left[ \int_{\max\{c_1(v), c_2(v)\}}^{\infty} \frac{1}{2\pi\sqrt{1-\rho_{XZ}^2}} \times \exp\left\{-\frac{1}{2(1-\rho_{XZ}^2)}[u^2 - 2\rho_{XZ}uv + v^2 - 2(1-\rho_{XZ}^2)t\sigma_Z v]\right\} \partial u \right] \partial v \quad (\text{A7})$$

To simplify this integral we substitute

$$w = \frac{u - \rho_{XZ}v}{\sqrt{1-\rho_{XZ}^2}} \quad \text{and} \quad s = v - t\sigma_Z. \quad (\text{A8})$$

and the new borders of integration become

$$c_3(s, t) = \frac{\frac{a - \mu_X}{\sigma_X} - \rho_{XZ}(s + t\sigma_Z)}{\sqrt{1-\rho_{XZ}^2}}$$

$$\text{and} \quad c_4(s, t) = \frac{\frac{b + (\mu_Z + \sigma_Z(s + t\sigma_Z)) - \mu_X}{\sigma_X} - \rho_{XZ}(s + t\sigma_Z)}{\sqrt{1-\rho_{XZ}^2}}.$$

(A9)

Now (A6) becomes:

$$e^{t\mu_Z + \frac{1}{2}\sigma_Z^2 t^2} \int_{-\infty}^{+\infty} \phi(s) \left[ \int_{\max\{c_3(s, t), c_4(s, t)\}}^{\infty} \phi(w) \partial w \right] \partial s \quad (\text{A10})$$

$$\text{with } \max\{c_3(s, t), c_4(s, t)\} = \begin{cases} c_3(s, t) & \text{if } s < \frac{a - b - \mu_Z - t\sigma_Z^2}{\sigma_Z} \\ c_4(s, t) & \text{if } s \geq \frac{a - b - \mu_Z - t\sigma_Z^2}{\sigma_Z} \end{cases} \quad (\text{A11})$$

## APPENDIX B:

In this appendix I concentrate on the numerator ( $m_1(t)$ ) of the mgf.

$$\text{Let } m_1(t) = e^{t\mu_Z + \frac{1}{2}\sigma_Z^2 t^2} \int_{-\infty}^{+\infty} \phi(s) \left[ \int_{\max\{c_3(s,t), c_4(s,t)\}}^{\infty} \phi(w) \partial w \right] \partial s.$$

For the expectation I need the derivative of  $m_1(t)$  with respect to  $t$  and evaluated in  $t=0$ .

$$\begin{aligned} \frac{\partial m_1(t)}{\partial t} &= (\mu_Z + \sigma_Z^2 t) e^{t\mu_Z + \frac{1}{2}\sigma_Z^2 t^2} \int_{-\infty}^{+\infty} \left[ \int_{\max\{c_3(s,t), c_4(s,t)\}}^{\infty} \phi_2(w, s; 0) \partial w \right] \partial s \\ &+ e^{t\mu_Z + \frac{1}{2}\sigma_Z^2 t^2} \int_{-\infty}^{+\infty} \left[ -\frac{\partial \max\{c_3(s,t), c_4(s,t)\}}{\partial t} \times \phi_2(\max\{c_3(s,t), c_4(s,t)\}, s; 0) \right] \partial s \quad (\text{B1}) \end{aligned}$$

$$\text{Define: } c(t) = \frac{a - b - \mu_Z - t\sigma_Z^2}{\sigma_Z}$$

(B3)

$$\begin{aligned} \frac{\partial m_1(t)}{\partial t} &= (\mu_Z + \sigma_Z^2 t) e^{t\mu_Z + \frac{1}{2}\sigma_Z^2 t^2} \int_{-\infty}^{+\infty} \left[ \int_{c_3(s,t)}^{\infty} \frac{1}{2\pi} \times \exp\left\{-\frac{1}{2}[w^2 + s^2]\right\} \partial w \right] \partial s \\ &+ (\mu_Z + \sigma_Z^2 t) e^{t\mu_Z + \frac{1}{2}\sigma_Z^2 t^2} \int_{c(t)}^{+\infty} \left[ \int_{c_4(s,t)}^{\infty} \frac{1}{2\pi} \times \exp\left\{-\frac{1}{2}[w^2 + s^2]\right\} \partial w \right] \partial s \\ &+ e^{t\mu_Z + \frac{1}{2}\sigma_Z^2 t^2} \int_{-\infty}^{c(t)} \left[ -\frac{\partial c_3(s,t)}{\partial t} \frac{1}{2\pi} \times \exp\left\{-\frac{1}{2}[(c_3(s,t))^2 + s^2]\right\} \right] \partial s \\ &+ e^{t\mu_Z + \frac{1}{2}\sigma_Z^2 t^2} \int_{c(t)}^{+\infty} \left[ -\frac{\partial c_4(s,t)}{\partial t} \frac{1}{2\pi} \times \exp\left\{-\frac{1}{2}[(c_4(s,t))^2 + s^2]\right\} \right] \partial s \quad (\text{B4}) \end{aligned}$$

with

$$\frac{\partial c_3(s,t)}{\partial t} = \frac{-\rho_{XZ}\sigma_Z}{\sqrt{1-\rho_{XZ}^2}}, \text{ and } \frac{\partial c_4(s,t)}{\partial t} = \frac{\frac{\sigma_Z^2}{\sigma_X} - \rho_{XZ}\sigma_Z}{\sqrt{1-\rho_{XZ}^2}}$$

$$(c_3(s,t))^2 + s^2 \Big|_{t=0} = \left( \frac{s - \frac{a - \mu_X}{\sigma_X} \rho_{XZ}}{\sqrt{1 - \rho_{XZ}^2}} \right)^2 + \left( \frac{a - \mu_X}{\sigma_X} \right)^2$$

$$(c_4(s,t))^2 + s^2 \Big|_{t=0} = \left( \frac{s - \frac{b - \mu_Y}{\sigma_Y} \times \frac{\rho_{XZ} \sigma_X - \sigma_Z}{\sigma_Y}}{\frac{\sigma_X}{\sigma_Y} \sqrt{1 - \rho_{XZ}^2}} \right)^2 + \left( \frac{b - \mu_Y}{\sigma_Y} \right)^2$$

Evaluated at t=0:

$$\frac{\partial m_1(t)}{\partial t} \Big|_{t=0} = (\mu_Z) \int_{-\infty}^{c(0)} \phi(s) \Phi(-c_3(s,0)) \partial s + (\mu_Z) \int_{c(0)}^{+\infty} \phi(s) \Phi(-c_4(s,0)) \partial s$$

$$+ \rho_{XZ} \sigma_Z \phi \left( \frac{a - \mu_X}{\sigma_X} \right) \Phi \left( \frac{c(0) - \frac{a - \mu_X}{\sigma_X} \rho_{XY}}{\sqrt{1 - \rho_{XZ}^2}} \right)$$

$$+ \frac{\sigma_X}{\sigma_Y} \left( \rho_{XZ} \sigma_Z - \frac{\sigma_Z^2}{\sigma_X} \right) \phi \left( \frac{b - \mu_Y}{\sigma_Y} \right) \Phi \left( - \frac{c(0) - \frac{b + \mu_Z - \mu_X}{\sigma_Y} \times \frac{\rho_{XZ} \sigma_X - \sigma_Z}{\sigma_Y}}{\frac{\sigma_X}{\sigma_Y} \sqrt{1 - \rho_{XZ}^2}} \right)$$

$$\frac{\partial m_1(t)}{\partial t} \Big|_{t=0} = (\mu_Z) \int_{-\infty}^{c(0)} \phi(s) \Phi(-c_3(s,0)) \partial s + (\mu_Z) \int_{c(0)}^{+\infty} \phi(s) \Phi(-c_4(s,0)) \partial s$$

$$+ (\sigma_X - \rho \sigma_Y) \phi \left( \frac{a - \mu_X}{\sigma_X} \right) \Phi \left( \frac{\rho \left( \frac{a - \mu_X}{\sigma_X} \right) - \left( \frac{b - \mu_Y}{\sigma_Y} \right)}{\sqrt{1 - \rho^2}} \right)$$

$$+ (\rho \sigma_X - \sigma_Y) \phi \left( \frac{b - \mu_Y}{\sigma_Y} \right) \Phi \left( \frac{\rho \left( \frac{b - \mu_Y}{\sigma_Y} \right) - \left( \frac{a - \mu_X}{\sigma_X} \right)}{\sqrt{1 - \rho^2}} \right).$$

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